AN INTRODUCTION TO THE PROBLEM OF PHOTOGRAPHING ARTIFICIAL SATELLITES

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Some basic information useful for obtaining photographic recordings of celestial bodies, especially artificial satellites, is presented in this paper for the benefit of those who are not very familiar with astronomy. In the present space age this information may be quite helpful to those who are not trained in astronomy, but who are working in related fields. The historical origin of the magnitude system is discussed and basic equations are developed that will permit calculation of an approximate value of the exposure time necessary for different situations. This will give, for example, those who employ electronic detection methods a means of determining the order of photographic exposure time so that the speed made possible by electronic methods may be compared with the conventional photographic speed.

THE MAGNITUDE SYSTEM

The earliest record of classification of stars according to apparent brightness appeared in 137 A.D. in Claudius Ptolemy's *Almagest*. The people of those days and earlier divided the visible stars into six groups which they called magnitudes. The twenty most brilliant stars were placed in magnitude one, and the faintest stars that they could see were placed in magnitude six. It is not known why six magnitudes were chosen, rather than some other number. After the invention of the telescope and the consequent discovery of celestial bodies fainter than sixth magnitude the system was extended to include these. There was no agreement as to scale at first and the magnitudes assigned to the telescopic bodies by the various observers differed enormously. Sir William Herschel's twentieth magnitude is about equal to our present day fourteenth magnitude. About 1830 Sir John Herschel, using a photometer, ascertained that the visible radiation received on earth from a first magnitude star is very nearly one hundred times as intense as the radiation received from a sixth magnitude star. He also announced that, although the magnitude scale is an arithmetic progression, the scale of apparent brightness associated with it seemed to be a geometric progression of the form

$$r = k^x \tag{1}$$

in which r is the ratio of apparent brightness between the two stars, k is a constant, and the variable x is the difference in the apparent magnitudes of the two stars. In 1856 N. Pogson in England suggested that John Herschel's findings be adopted and that the scale used for expressing the magnitude be such that a first apparent magnitude star will provide exactly one hundred times as much illumination on the earth as a sixth magnitude star. Eq. (1) then becomes

$$100 = k^{6-1} = k^5. \tag{2}$$

In logarithmic form this equation is

$$2 = 5 \log k, \tag{3}$$

hence

$$\log k = 0.4 \tag{4}$$

$$k = 2.512. \tag{5}$$

This value of k is accepted now as the ratio of flux of light received from two celestial bodies where the difference in apparent magnitude is one, and for any difference $n-m$

$$r=(2.512)^{n-m}=10^{0.4\,(n-m)}. \tag{6}$$

In this, m is the apparent magnitude of the brighter celestial body and n is that of the fainter celestial body. Zero or negative values of apparent magnitude are assigned to objects brighter than the standard first apparent magnitude star. The apparent magnitude of the sun is -26.72 and that of the faintest star recorded with the Hale telescope on Palomar Mountain is about $+23$ (Mehlin, 1959).

Since the apparent magnitude expresses only the flux of light (total number of quanta of light = photons) received from a source and does not take into account the angular size of a celestial body, the total flux of photons coming from a star will be concentrated in the focal plane of a long focal length telescope onto a smaller area than the flux of photons coming from the moon or planet will be. Consequently, if using a long focal length telescope for recording the image of a star and of a planet of the same apparent magnitude, the flux of photons from the star will be concentrated on a smaller number of photographic crystallites than the photons from the planet and the exposure time necessary for recording the image of the star will be less than that needed for the planet. The apparent magnitude may refer to any spectral band; the apparent visual magnitude is based on the spectral response of the human eye; other expressions are necessary and used for photo-sensors that have a different spectral response.

CONVERSION OF THE APPARENT MAGNITUDE INTO FLUX OF PHOTONS

A conversion from apparent magnitude into photon density per unit of receptor area and time must be established for calculating an approximation of the theoretical limit for detection. The sun will be taken as a standard and the results can be applied to all bodies radiating energy with nearly the same spectral distribution. The solar energy per square centimeter per second, and its conversion into the number of photons per square millimeter per second, that reaches the outer portion of the earth's atmosphere at normal incidence are shown in table 1 by bands from $\lambda 0.29$ to 1.45μ. This table is taken directly from Gebel's publication "Daytime Detection of Stars" (Gebel, 1958). An extension from $\lambda 1.45$ to 5μ by Wylie is shown in table 2. The tables were computed from the work of Parry Moon, who summarized all available information on the intensity of solar radiation as received at the earth's outer atmosphere at normal incidence, and published a table giving its intensity in watts per square meter per micron from 0.29μ to 5μ (Moon, 1940). At a shorter wavelength than about 0.29μ the solar energy is absorbed by our atmosphere. Gebel's table indicates that between 0.29μ and 1.45μ the total number of quanta of solar light Q per square millimeter per second arriving at the outer atmosphere at normal incidence is

$$Q\approx 41.9\times 10^{14}. \tag{7}$$

Between 0.29μ and 1.45μ the average number of photons Q_o per square millimeter per second received at the outer atmosphere of the earth at normal incidence from a source of the same spectral distribution as our sun, and of apparent magnitude zero is

$$Q_o\approx Q\times 2.512^{-26.7} \tag{8}$$

or

$$Q_o\approx 42\times 10^{14}\times 10^{-(0.4\times 26.7)}\approx 8.7\times 10^{4}. \tag{9}$$

In the same range the average number of photons Q_n per square millimeter per second received at the earth's outer atmosphere at normal incidence from a celestial body of any apparent magnitude n with the same spectral distribution as our sun is

$$Q_n\approx 8.7\times 10^{(4-0.4n)}. \tag{10}$$

TABLE 1

*Energy/cm² from sun; quanta/mm² from sun by bands 0.29–1.45μ**

Spectral band $\lambda_1 \rightarrow \lambda_2$ $\lambda(m\mu)$	Energy flux per unit bandwidth $\frac{G\lambda_1+G\lambda_2}{2}$ $G\left(\frac{erg}{sec\ cm^2\ m\mu}\right)$	Quantum flux per unit bandwidth $\frac{(G)(\lambda)}{hc}=\frac{G(\lambda)}{1.986}\times 10^8$ $Q_i\left(\frac{quanta}{mm^2\ sec\ m\mu}\right)$	Quantum flux per band $(\lambda_2-\lambda_1)Q\lambda$ $Q_i\left(\frac{quanta}{mm^2\ sec}\right)$
290–291	95	1.39×10^{11}	1.39×10^{11}
291–292	220	3.23	3.23
292–293	269.5	3.97	3.97
293–294	303.5	4.48	4.48
294–295	331.5	4.91	4.91
295–296	356.5	5.30	5.30
296–297	378.5	5.65	5.65
297–298	399.5	5.98	5.98
298–299	420	6.31	6.31
299–300	440	6.64	6.64
300–301	460	6.96	6.96
301–302	479.5	7.28	7.28
302–303	498	7.59	7.59
303–304	515.5	7.88	7.88
304–305	532	8.16	8.16
305–306	548	8.43	8.43
306–307	563.5	8.70	8.70
307–308	578.5	8.96	8.96
308–309	593.5	9.22	9.22
309–310	608.5	9.48	9.48
310–311	622	9.72	9.72
311–312	634	9.94	9.94
312–313	646	1.017×10^{12}	1.017×10^{12}
313–314	658	1.039	1.039
314–315	670	1.061	1.061
315–316	681	1.082	1.082
316–317	691	1.101	1.101
317–318	701	1.121	1.121
318–319	711	1.140	1.140
319–320	721	1.160	1.160
			$\lambda_2-\lambda_1=5m\mu$
320–325	744	1.208	6.04×10^{12}
325–330	779	1.285	6.43
330–335	811	1.358	6.78
335–340	841	1.430	7.15
340–345	871	1.502	7.51
345–350	901	1.577	7.88
			$\lambda_2-\lambda_1=10m\mu$
350–360	946	1.691	1.691×10^{13}
360–370	1011	1.858	1.858
370–380	1084	2.047	2.047
380–390	1161	2.251	2.251
390–400	1253	2.492	2.492
400–410	1516	3.091	3.091
410–420	1747	3.65	3.65
420–430	1777	3.80	3.80
430–440	1864	4.08	4.08
440–450	1988	4.46	4.46
450–460	2066	4.73	4.73
460–470	2108	4.94	4.94
470–480	2123	5.08	5.08
480–490	2115	5.17	5.17

TABLE 1—*Continued*

Spectral band $\lambda_1 \to \lambda_2$ $\lambda(m\mu)$	Energy flux per unit bandwidth $\frac{G\lambda_1+G\lambda_2}{2}$ $G\left(\frac{erg}{sec\ cm^2\ m\mu}\right)$	Quantum flux per unit bandwidth $\frac{(G)(\lambda)}{hc}=\frac{G(\lambda)}{1.986}\times10^8$ $Q_i\left(\frac{quanta}{mm^2\ sec\ m\mu}\right)$	Quantum flux per band $(\lambda_2-\lambda_1)Q\lambda$ $Q_i\left(\frac{quanta}{mm^2\ sec}\right)$
490–500	2082	5.19×10^{12}	5.19×10^{13}
500–510	2031	5.16	5.16
510–520	1977	5.13	5.13
520–530	1933	5.11	5.11
530–540	1903	5.13	5.13
540–550	1886	5.18	5.18
550–560	1870	5.23	5.23
560–570	1851	5.27	5.27
570–580	1830	5.30	5.30
580–590	1807	5.32	5.32
590–600	1779	5.33	5.33
600–610	1744	5.31	5.31
610–620	1709	5.29	5.29
620–630	1672	5.26	5.26
630–640	1635	5.23	5.23
640–650	1598	5.19	5.19
650–660	1561	5.15	5.15
660–670	1526	5.11	5.11
670–680	1490	5.06	5.06
680–690	1456	5.02	5.02
690–700	1422	4.98	4.98
700–710	1388	4.93	4.93
710–720	1354	4.87	4.87
720–730	1321	4.82	4.82
730–740	1287	4.76	4.76
740–750	1253	4.70	4.70
			$\lambda_2-\lambda_1=0.05\mu$
750–800	1167	4.55	2.28×10^{14}
800–850	1037	4.31	2.15
850–900	924	4.07	2.04
900–950	826	3.85	1.92
950–1μ	744	3.65	1.825
			$\lambda_2-\lambda_1=0.1\mu$
1μ –1.1μ	648	3.43	3.43×10^{14}
1.1–1.2	539	3.12	3.12
1.2–1.3	442	2.78	2.78
1.3–1.4	357	2.43	2.43
1.4–1.45	303	2.17	1.09

Total for λ's 0.29–1.45μ (detectable by photo emitters) $=11.6\times10^5$ erg/cm² sec $=4.192\times10^{17}$ quanta/cm² sec.

*Values for computation of Q_i from Parry Moon, J. Franklin Inst., Nov. 1940.

The average number of photons Q_t arriving per second at the focal plane of any imaging system from a celestial body is

$$Q_t = Q_n A\eta \tag{11}$$

where A is the effective light collecting area of the imaging system in square millimeters, and η is the factor that takes into consideration all the losses of light

TABLE 2

Appendix to table 1 for bands 1.45–5μ

Spectral band	Energy flux per unit bandwidth	Quantum flux per unit bandwidth	Quantum flux per band
$\lambda_1 \rightarrow \lambda_2$	$\frac{G\lambda_1 + G\lambda_2}{2}$	$\frac{(G)(\lambda)}{hc} = \frac{G(\lambda)}{1.986} \times 10^8$	$(\lambda_2 - \lambda_1)Q\lambda$
$\lambda(m\mu)$	$G\left(\frac{erg}{sec\ cm^2\ m\mu}\right)$	$Q_i\left(\frac{quanta}{mm^2\ sec\ m\mu}\right)$	$Q_i\left(\frac{quanta}{mm^2\ sec}\right)$
1.45–1.5	270	2.110×10^{12}	1.055×10^{14}
1.5–1.6	237	1.848	1.848
1.6–1.7	195.5	1.623	1.623
1.7–1.8	162.5	1.431	1.431
1.8–1.9	136.0	1.266	1.266
1.9–2.0	114.5	1.124	1.124
2.0–2.1	97.15	1.002	1.002
2.1–2.2	82.85	0.896	0.896
2.2–2.3	71.10	0.805	0.805
2.3–2.4	61.35	0.726	0.726
2.4–2.5	53.20	0.656	0.656
2.5–2.6	46.35	0.595	0.595
2.6–2.7	40.55	0.541	0.541
2.7–2.8	35.65	0.494	0.494
2.8–2.9	31.45	0.451	0.451
2.9–3.0	27.80	0.413	0.413
3.0–3.1	24.70	0.379	0.379
3.1–3.2	22.05	0.350	0.350
3.2–3.3	19.70	0.322	0.322
3.3–3.4	17.60	0.297	0.297
3.4–3.5	15.75	0.273	0.273
3.5–3.6	14.20	0.254	0.254
3.6–3.7	12.85	0.236	0.236
3.7–3.8	11.65	0.220	0.220
3.8–3.9	10.60	0.205	0.205
3.9–4.0	9.65	0.192	0.192
4.0–4.1	8.82	0.180	0.180
4.1–4.2	8.095	0.169	0.169
4.2–4.3	7.425	0.159	0.159
4.3–4.4	6.800	0.149	0.149
4.4–4.5	6.215	0.139	0.139
4.5–4.6	5.685	0.130	0.130
4.6–4.7	5.215	0.122	0.122
4.7–4.8	4.805	0.115	0.115
4.8–4.9	4.470	0.109	0.109
4.9–5.0	4.190	0.104	0.104

Total for λ's 1.45–5μ (detectable by photo conductors) $= 1.6 \times 10^5$ ergs/cm² sec $= 1.942 \times 10^{17}$ quanta/cm² sec.

*Values for computation of Q_i from Parry Moon, J. Franklin Inst., Nov. 1940.

between the outer atmosphere and the focal plane. Substituting for Q_n in Eq. (11) it's value from Eq. (10) results in

$$Q_t \approx 6.8 \times 10^{(10-0.4n)} a^2 \eta \tag{12}$$

where a is the useful aperture of the imaging system in meters. An approximation for the number of photons Q_v per second in the visible portion of the spectrum (0.41μ to 0.67μ) from a source with a spectral distribution similar to the sun arriving at the focal plane may be derived in a similar manner; it is

$$Q_v \approx 2.1 \times 10^{(10-0.4n)} a^2 \eta. \tag{13}$$

BASIC CONSIDERATIONS FOR THE EXPOSURE TIME

The image of a point source of light cannot be represented by a single photographic grain because many of the silver halide crystallites suspended in a photographic emulsion, even though not exposed to light at all, are developed randomly into grains (fog). Sky background radiation may cause additional fogging and is a principal factor in limiting the useful duration of astronomical photographic exposure times. Hence, the photographic image of a star cannot be represented by a single grain, but must be expressed by the concentrated configuration formed by many grains which can be discriminated against the grain density of the fog.

A large amount of the sky background radiation may be due to terrestrial sources but in a favorable location it may be mainly due to such sources as diffused light from celestial bodies and a permanent auroral luminosity that overspreads the sky both day and night. The lowest illumination caused by the cloudless night sky is approximately 3×10^{-5} ft-c (Natural Illumination Charts, Department of the Navy, 1952). Since 1ft-c of visible sunlight is approximately 10^{11} photons/mm² sec for 0.41μ to 0.67μ the night sky illumination corresponds to 3×10^{6} photons/mm² sec, neglecting any difference in color temperature.

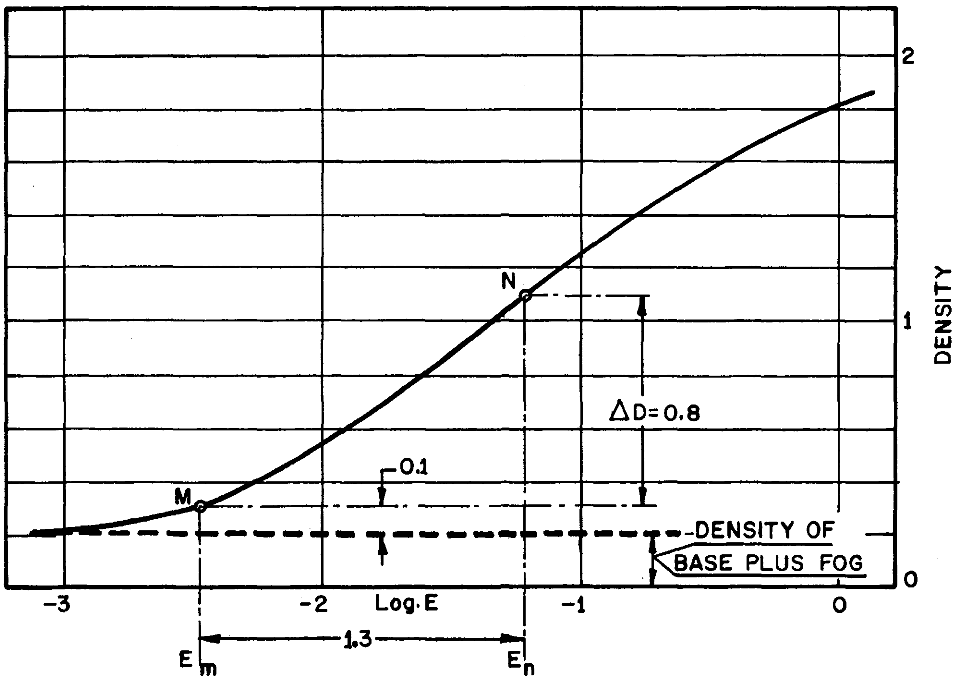

FIGURE 1. Illustration for determining photographic speed as defined by the American Standard Method.

For comparative purposes the sensitivity of conventional photographic emulsions is usually expressed by a number, the ASA speed (American Standard Method for Determining Speed of Photographic Negative Materials, April 1960). The arithmetic ASA number is the ratio between the number 0.8 and the exposure E_M in meter-candle-seconds necessary to produce a density of 0.1 above the density D_{BF} of base plus fog. For this a picture gamma curve (fig. 1) is used which goes through the point M which corresponds to E_M and another point N which corresponds to $E_M+1.3$ on the exposure axis and a point on the density axis with a value of $D_{BF}+0.9$. The color temperature of the light source for this procedure is 2850° K and filters are necessary as specified by the American Standard Association. For some purposes the speeds of photographic emulsions are compared by the value of the "relative sensitivity" per meter-candle-second if exposed to tungsten light, and referring to a density that is 0.6 above the density D_{BF} for the recommended time of development and kind of developer used. The density D_{BF} may be as high as 0.4 but usually is in the order of 0.1 for many emulsions and for some kinds of developers. For spectroscopic photographic emulsions the ASA speed index would offer little usefulness and hence characteristics expressing the fundamental relationships are better used.

The relationship between the density-light flux characteristic and the ASA index is complicated because of the gamma curve involved in the ASA index and the correct relating factor must be determined for each situation from experimental data. For example, the Kodak Data Book lists a density of 0.1 above D_{BF} for Kodak Panatomic X Film having a speed of ASA Daylight 25 if exposed to a radiation of 0.018 m-c-sec assuming optimum exposure; and for the same conditions Kodak Tri-X Film having an ASA Daylight 200 has a density of 1.0 above D_{BF}. Since a photographic image is made up of individual grains the situation exists that wherever a grain has been developed the film is practically nontransparent, and where there are no grains the film is almost completely transparent. Thus for the two films and conditions mentioned, the total number of grains is enough so that about 20 and 90 percent of a unit area is covered, respectively. The Panatomic Film has about 2.5 times the resolving power as the Tri-X which indicates that the average projected diameter of the grains of the two emulsions differs also by about this factor. Since the percentage of unit area covered differs by a factor 4.5 it must be concluded that the total number of grains developed in both cases does not differ very much. The difference in density therefore was mostly the result of the difference in grain size.

For the present state of the art in manufacturing photographic emulsions it seems to be unavoidable that those emulsions that have the highest ASA index have also the largest grain size and therefore have the fewest number of grains per unit of area. Consequently, they are the poorest in resolution. Grains in ultrafast photographic emulsions, developed for an index of 1000 ASA, have an average projected diameter in the order of 5μ, while the grains developed in emulsions used for nuclear work are only in the order of 0.2μ in average projected diameter. Developed microfilm has grains with an average projected diameter in the order of 0.03μ, but the speed is only in the order of 0.025 ASA. It appears that the ratio of the ASA index of the different photographic emulsions to each other is roughly proportional to the average projected area of the grains. Since the transparency of a developed photographic emulsion mostly depends on the projected total area of all the grains, it can be concluded from the foregoing that the average mathematical value of the effective quantum efficiency ξ (which is the reciprocal of the average number of photons focused onto the emulsion to the number of useful grains produced) is nearly constant for the different photographic emulsions under the conditions for which the ASA speed number is computed.

The effective average value of the quantum efficiency ξ is a function of the lightflux density, spectral distribution and bandwidth, and of the number of crystallites already activated. At a very low flux of light the quantum efficiency is reduced by failure of the reciprocity law and if computed for a high density of the photographic emulsion the value is also reduced because many photons will fall on crystallites already activated by previous photons. In the following an approximate value of the quantum efficiency will be determined for Tri-X Roll Film for a density for which the ASA speed was determined. From the density-log exposure characteristic we find that a logarithm of exposure of minus 2.75 produces the necessary change in the density of 0.1 above D_{BF} as required for the ASA speed. Assuming sunlight for the exposure and for 1 m-c about 10^{10} photons/mm^2 sec (0.41μ to 0.67μ) one may write

$$\xi \approx \frac{1-\dfrac{1}{O_F}}{SL\ 10^{10}} \tag{14}$$

where S is the average of the projected grain size in square millimeters, O_F is the change of the opacity caused by the exposure and L is the flux of light used for the exposure in meter-candle-seconds. In the example for a grain size of about 10^{-5} square millimeters which usually is obtained under such conditions with Tri-X Roll Film

$$\xi \approx \frac{1-\dfrac{1}{1.21}}{1.8\times10^{-3}\times10^{-5}\times10^{10}} \approx 10^{-3}. \tag{15}$$

Even though this is an approximation, conventional photographic emulsions have a quantum efficiency which is of this order and no photographic emulsion has been developed yet which has a quantum efficiency substantially larger than this value. It seems, as previously expressed, that the difference in speed for different photographic emulsions is not due to the development of a substantially different number of grains for the same light flux but rather that the difference in the opacity of the different emulsions is mostly controlled by the effective average diameter of the grains. The factor Z, by which the exposure time has to be extended to compensate for the failure of the reciprocity law, with Tri-X Film, between a logarithm of exposure of -1 and -4, is about 10 and for the Kodak 103a–O emulsion between logarithm of exposure -2 and -5 is about 1.5.

The process by which the energy of the photons is used for producing grains is random. Since the countable grains due to sky illumination and to the inherent background of the photographic emulsion are randomly distributed, determinations of deviations in the number of useful grains obtained per unit area may be made by statistical mathematics.

For example, a star image that is assumed to be 0.025 mm in diameter could cover a maximum number of about 25 usable grains of a developed photographic emulsion in which the grains have an average projected diameter of 5μ. If a probability of 0.95 in certainty of detection of the star image is demanded, the area covered by it should contain in addition to the background grains approximately five times the standard deviation in the number of background grains as in an equal area containing only background grains (Gebel 1961). If the area covered by the image of the celestial body is divided into many arbitrarily chosen elements of resolution this conclusion is valid for each element of resolution. If, for each element of resolution, g represents the average number of background grains developed and σ represents the standard deviation in the average number of background grains, it is customary to assume that

$$\sigma = g^{1/2}. \tag{16}$$

If C represents the average number of photographic grains resulting from the radiation from the celestial body and C' the value for obtaining a probability of 0.95 for detection, one may write

$$C' \approx 5\sigma \approx 5g^{1/2}. \tag{17}$$

Thus, for example, if there is an average of nine background grains per resolution element the standard deviation of the background grains is 3; then the resolution elements covered by the star image need 3×5, or 15 additional grains to obtain a probability of 0.95 for detection. Therefore, in the detection of celestial bodies which cause only a small number of photographic grains several photographs of the same region are usually taken for comparison purposes. In practice, with long focal length telescopes, the image diameter is often about 0.1 mm, and for such an area approximately 100 useful grains are generally developed, making visual discrimination of the image of the celestial body against the background grains easily possible. Thus, as explained earlier, assuming a quantum efficiency of 10^{-3}, about 100,000 quanta of light are required for the above condition.

The quantity C during the exposure time t in seconds may be derived from

$$C \approx \xi Q_v t \tag{18}$$

where by using this equation the valid restrictions of ξ as previously explained should be considered. Making use of Eq. (13) for Q_v and solving for the exposure time t

$$t \approx \frac{5\times 10^{(0.4n-11)} C}{a^2 \eta\ \xi}. \tag{19}$$

In accordance with previous statements for the condition found for any ASA number, one may write as an approximation for the maximum number of useful grains Ω that can be developed for each square millimeter of present conventional emulsions before saturation is reached, which means that transparency of the photographic record is practially lost,

$$\Omega \approx \frac{1}{25\times 10^{-6}} \times \frac{1000}{R} \approx \frac{4\times 10^7}{R} \tag{20}$$

where R is the ASA film speed, and as reference an ASA speed of 1000 with an emulsion having grains with an average projected diameter of 25×10^{-6} mm² is used. The approximation for the maximum number ω of useful grains that can occur for an area with a diameter d in millimeters by using Eq. (20) is

$$\omega \approx \frac{10^7\ \pi\ d^2}{R} \geqq 1 \tag{21}$$

in which d is the effective diameter of the area onto which the image of the celestial body is focused. The maximum number of photons Q_e needed for optimum density for an area with a diameter d by using Eq. (18) analogously is

$$Q_e \approx \frac{\omega}{\xi} \tag{22}$$

where the restrictions for ξ mentioned before have to be taken into consideration, i.e. failure of reciprocity, etc.

As previously discussed, by making certain assumptions a practical value for the quantum efficiency ξ of most present conventional photographic emulsions, neglecting failure of the reciprocity law, is about 0.001 and one may substitute this value in Eq. (22) wherever it safely applies. Then by using Eq. (21)

$$Q_e \approx \frac{10^7\ \pi\ d^2}{10^{-3} R} \approx \frac{10^{10}\ \pi\ d^2}{R}. \tag{23}$$

Let the optimum exposure time T in seconds be that time for which practically all the useful grains in an area with the diameter d are developed as a result of the radiation from the celestial body. When Eq. (23) is divided by Eq. (13), then

$$T \approx \frac{10^{10}\ \pi\ d^2}{Q_v R} \approx \frac{1.5\times 10^{0.4n}\ d^2}{R\ a^2 \eta}. \tag{24}$$

It has to be understood that this is an approximation only and has been derived here for the purpose of obtaining a reference value from which by taking further trial exposures the correct value for each condition can be determined. For extended exposure times, using the factor Z to compensate for the failure of the reciprocity law yields a corrected exposure time T_R which is

$$T_R \approx T\ Z. \tag{25}$$

Example 1. A 10-inch telescope is used with a film having an effective speed of 1000 ASA, and the combined transmission efficiency of the telescope and the atmosphere is 0.5. The diameter over which the star image scintillates is approximatly 0.3 mm, because of seeing conditions and the focal length of the telescope. Then using Eq. (24) we find as an approximation for the maximum exposure time for a star of 10th apparent magnitude

$$T \approx \frac{1.5 \times 10^{0.4n} \times 0.3^2}{1000 \times 0.25^2 \times 0.5} \approx 43 \text{ sec.}$$

For this exposure time and the particular emulsion used, the manufacturer quotes a factor Z of about 1.25. Hence

$$T_R \approx 43 \times 1.25 \approx 54 \text{ sec.}$$

THE PROPER FOCAL LENGTH FOR A MOVING TARGET

The preceding discussion assumes a source of apparent fixed position, so that all the photons received from it would be focused on a single spot of the photo-sensor. The moving target case, such as photography of an artificial satellite, has been discussed by Gebel in ASTIA Document AD 210752 (Gebel, 1959). The effective time the image of a moving target will remain on the individual resolution areas of a photographic emulsion will depend on the speed of the image of the target in the focal plane of the telescope, and this speed depends on the apparent angular speed of the moving target and the focal length of the telescope. Consequently, the effective exposure time is a function of the focal length, the diameter of the objective lens (its light gathering power), the velocity, apparent magnitude and effective angular size of the moving target, and the speed of the photographic emulsion used. The focal length to be used for making recognizable recordings cannot exceed a certain value because the relative speed with which the image of the celestial body moves over the photographic emulsion must be slow enough to permit activation of a sufficient number of photographic crystallites to produce an ample density per unit of area in the developed emulsion.

An approximation for the time during which the image of the moving target effectively remains on a single element of resolution of the photographic emulsion must be found. The effective linear diameter d in millimeters of the image at the focal plane of the imaging system is given by

$$d = 4.85 \times 10^{-3} F\, \theta \tag{26}$$

where F is the focal length of the imaging system in meters, θ is the apparent angular diameter in seconds of arc of the moving target which is the angular diameter enlarged above the true angular diameter as determined by the true size and distance of the target. The enlargement is due to scintillation of the air, scattering or haze, loss of resolution by the telescope or photo-sensor, etc.

Let θ_Δ be the angular speed in seconds of arc per second of time as determined by the movement of the target, and let V be the corresponding linear velocity of the object relative to the observer in meters per second. Let D be the distance of the target from the observer in meters, then

$$\theta_\Delta = \frac{10^6\, V}{4.85\, D}\,. \tag{27}$$

An approximation for the time t during which the image of the moving target is moving over an area corresponding to the diameter d may be found from the relationship

$$t \approx \frac{\theta}{\theta_\Delta} \tag{28}$$

from which it follows that

$$t \approx \frac{10^{-3} D\, d}{F\, V}. \tag{29}$$

Setting Eq. (29) equal to Eq. (24) by assuming $t=T$ and $F=F_m$, where F_m is the longest permissible focal length for obtaining practically all the useful photographic grains in the image area, and by solving for F_m the result is:

$$F_m \approx \frac{6.7 \times 10^{-(4+0.4n)} D\, R\, a^2 \eta}{Vd}. \tag{30}$$

When d is replaced by it's value from Eq. (26),

$$F_m \approx \left(\frac{1.4 \times 10^{-(1+0.4n)} D\, R\, a^2 \eta}{\theta V}\right)^{1/2}. \tag{31}$$

When attempting photography of a meteor, it should be realized that it is not the original meteoric particle that is seen but the luminous trail produced. The meteors consist mostly of iron or stone and practically all are less than 1 mm in diameter. They enter the earth's atmosphere with a speed of about 35 km/sec and friction with the air at a height of about 110 km is sufficient to start vaporization. The height at which vaporization is completed depends on the size of the meteor and for the average size is at about 90 km. The oblique luminous trail of the average meteor is about 40 km long and the duration is about 1.5 sec. Larger meteors may burst in the air and drop fragments on the earth, and still larger ones may come through without bursting. The air just ahead of the meteor is rapidly compressed. This causes intense heating and ionization of the molecules involved and radiant energy is emitted. The resultant glowing column, streaming behind the meteor, is narrow on formation, but dissipates rapidly and at a distance of 175 km from the observer would be at least 2 sec of arc in width 1 sec after formation. Since radiant energy comes from all parts of the trail and not just the moving head, Eqs. (30) and (31) would not apply to meteors. The time duration and expansion of the trail is independant of the motion of the head and not related to the linear velocity V or the angular speed θ as used in Eqs. (30) and (31).

Example 2. The rocket of an artificial satellite is to be photographed with a telescope of 25-cm aperture when the rocket is at a distance from the observer of 525 km. The rocket is tumbling and the diameter of its projected area is estimated to be 50 m. The orbital speed is 20,000 miles/hr=8.9 km/sec. Its visual magnitude is about zero. The overall transmission efficiency of telescope and atmosphere is judged to be 0.5. Film of 1000 ASA is to be used. The maximum focal length of the telescope is found by using Eq. (31) to be

$$F_m \approx \left(\frac{1.4 \times 10^{-(1+0.4\times 0)} \times 525000 \times 1000 \times 0.25^2 \times 0.5}{\dfrac{50}{525000 \times 4.85 \times 10^{-6}} \times 8900}\right)^{1/2} \approx 3.6 \text{ meters.}$$

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